

- Gibbard, A. (1969), "Intransitive Social Indifference and the Arrow Dilemma", mimeograph.
- Joguen, J. A. (1967), "L-fuzzy Sets", Journal of Mathematical Analysis and Applications, 18.
- Kolm, B. (1984), "Efficient and Binary Consensus Function on Transitively Valued Relations", Mathematical Social Sciences, 8, 46-61.
- Jurmi, H. (1981), "Approaches to Collective Decision Making with Fuzzy Preference Relations", Fuzzy Sets and Systems, 6, 249-259.
- Ovchinnikov, S. V. and V. M. Osernoy (1988), "Using Fuzzy Binary Relations for Identifying Non-Inferior Decision Alternatives", mimeograph.
- Sen, A. K. (1970), Collective Choice and Social Welfare. Amsterdam: North-Holland Publishing Company.
- Subramanian, S. (1987), "The Liberal Paradox with Fuzzy Preferences", Social Choice and Welfare, 4, 213-223.
- Tanaka, T. (1984), "Fuzzy Preference Orderings in Group Decision Making", Fuzzy Sets and Systems, 12, 117-131.

SINGLE-PEAKEDNESS IN WEIGHTED AGGREGATION OF FUZZY OPINIONS IN A FUZZY GROUP

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Abstract: In this paper it is considered a formal approach to the problem of aggregating individual opinions in a fuzzy group, when alternatives can be represented in a real hyper-space and each individual defines his/her fuzzy set of non rejectable alternatives. On one hand, weighted aggregation rule for consensus opinion is axiomatically justified. On the other hand, it is shown a sufficient condition for the stability of such consensus solution.

Keywords: aggregation rules, group decision making, fuzzy opinions.

1. INTRODUCTION

Every society is faced with the problem of opinion aggregation each time its individuals define different judgments or different preference attitudes. Amalgamating them into a consensus represents in this way a key point for the development of any group of persons. However, as shown by Arrow (1951, 1964), there is no general methodology for aggregating crisp individual preferences through a "social welfare function" satisfying some natural rationality conditions. Two ways for avoiding such a result have been proposed in the past: relaxing the concept of solution by considering, for example, "social choice functions" (Sen, 1970), or constraining the preferences domain of individual by assuming, for example, that such crisp preferences verify "single-peakedness" property (Black, 1958). This paper is based on the last approach, but allowing a continuous strength of preferences.

The three seminal books cited above suppose that each

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individual i defines a crisp binary preference relation R_i on the set of feasible alternatives X ($xP_i y$ will denote that $xR_i y$ holds but not $yR_i x$). Black's single-peakedness means that there exists a strict ordering S on X such that all individual preference orderings can be represented by a reference curve with only one peak (with one or two elements), in such a way that on each side of such a peak it slopes downwards: formally, for each individual i and all distinct alternatives $x, y, z \in X$ the relation $xP_i z$ holds whenever $xR_i y$ for some alternative y between x and z ($xySxz$ or $zSyxz$). Then it was shown that the method of majority decision leads to a consensory transitive ordering on the set of alternatives. Inada (1964) has pointed out that single-peakedness with respect to the entire set of alternatives is not necessary: single-peakedness with respect to every triple of alternatives is enough.

FORMAL DESCRIPTION OF THE MODEL

Let $\beta: G \rightarrow [0, 1]$ be a fixed fuzzy group of individuals (experts), G having two elements at least; $\beta(i) > 0$, $\forall i \in G$, is the degree of competence of individual i (see Cholewa, 1985, for a discussion of the concept of competence), and let us assume that the set of (at least two) feasible alternatives can be represented as a convex subset of the real hyper-space. This does not seem a serious restriction, since most practical problems are multidimensional in nature according to a finite number of single characteristics, each one usually represented in the real line, but it requires a continuous degree in such a set of feasible alternatives. Individuals will be supposed to give their opinions through a fuzzy reference, that is, a fuzzy set of non-rejectable alternatives $\mu_i: X \rightarrow [0, 1]$, where $\mu_i(x)$ represents the degree of membership of alternative x in the set of solutions, as given by individual i . Following Montero (1985), we must be able to define an aggregation rule, that is, a correspondence

$$*: (\mathcal{F}(X) \times \mathcal{P}(G)) \times (\mathcal{F}(X) \times \mathcal{P}(G)) \rightarrow (\mathcal{F}(X) \times \mathcal{P}(G))$$

which assigns to each pair of opinions $\mu_A, \mu_B \in \mathcal{F}(X)$ from two disjoint and non-empty groups $A, B \in \mathcal{P}(G)$ an opinion $\mu_{A \cup B} \in \mathcal{F}(X)$ of the union group $A \cup B \in \mathcal{P}(G)$, satisfying the associativity and commutativity:

$$((\mu_A, A) * (\mu_B, B)) * (\mu_C, C) = (\mu_A, A) * ((\mu_B, B) * (\mu_C, C))$$

$$(\mu_A, A) * (\mu_B, B) = (\mu_B, B) * (\mu_A, A)$$

for arbitrary disjoint and non-empty groups $A, B, C \in \mathcal{P}(G)$.

In this way, opinion μ_A of a group A will be represented by the pair (μ_A, A) and

$$(\mu_A, A) * (\mu_B, B) = (\mu_{A \cup B}, A \cup B) \quad \forall A, B \in \mathcal{P}(G), A \cap B = \emptyset$$

It can be assumed that the aggregated opinion $\mu_{A \cup B}$ depends on the members of both groups A and B through their sizes, $\text{card}(A)$ and $\text{card}(B)$, and their associated competence β_A and β_B , obtained by aggregating the individual competences. That is, by applying an associative and commutative correspondence $\theta: ([0, 1] \times \mathcal{P}(G)) \times ([0, 1] \times \mathcal{P}(G)) \rightarrow ([0, 1] \times \mathcal{P}(G))$

$$(\beta_A, A) \theta (\beta_B, B) = (\beta_{A \cup B}, A \cup B) \quad \forall A, B \in \mathcal{P}(G), A \cap B = \emptyset$$

which assigns an aggregated competence $\beta_{A \cup B} \in [0, 1]$ of the union group $A \cup B \in \mathcal{P}(G)$ of two disjoint and non-empty groups $A, B \in \mathcal{P}(G)$, and it can be assumed also that $\beta_{A \cup B}$ depends on A and B only through their competence (β_A and β_B , respectively) and their size ($\text{card}(A)$ and $\text{card}(B)$, respectively).

Obviously, input data of our aggregation problem must be all the individual opinions $(\mu_i, \beta_i)_{i \in G}$ and their associated individual competences $(\beta_i, \beta(i))_{i \in G}$, and both aggregation rules must be each one ethical and connected in a rational way (as shown in Montero, 1988a, not every opinion aggregation rule $*$ is compatible with any fixed competence aggregation rule θ). For example, the following conditions (i), (ii), (iii) and (i'), (ii'), (iii') can be easily accepted for θ and $*$, respectively:

(i) $\beta_{A \cup B} = \beta_A \cup \beta_B$ holds for any non-empty groups being

$A \cap B = A' \cap B' = \emptyset$ and such that

$$\text{card}(A) = \text{card}(A'), \beta_A = \beta_{A'}$$

$$\text{card}(B) = \text{card}(B'), \beta_B = \beta_{B'}$$

(ii) $\beta_{A \cup B} = \beta_B$ holds for any non-empty groups ($A \cap B = \emptyset$) such that $\beta_A = \beta_B$.

(iii) If $\beta_A \geq \beta_{A'}$ and $\beta_B \geq \beta_{B'}$ for two non-empty and disjoint groups A and B , then $\beta_{A \cup B} \geq \beta_{A' \cup B'}$.

(i') $\mu_{A \cup B} = \mu_A \cup \mu_B$ holds for any non-empty groups being $A \cap B = A' \cap B' = \emptyset$ and such that $\mu_A = \mu_{A'}$ and $\mu_B = \mu_{B'}$, and

$$\text{card}(A) = \text{card}(A'), \beta_A = \beta_{A'}$$

$$\text{card}(B) = \text{card}(B'), \beta_B = \beta_{B'}$$

(ii') $\mu_{A \cup B}(x) = \mu_B(x)$ holds for any non-empty groups ($A \cap B = \emptyset$) such that $\mu_A(x) = \mu_B(x)$.

(iii') If $\mu_A(x) \geq \mu_{A'}(x)$ and $\mu_B(x) \geq \mu_{B'}(x)$ hold for two non-empty and disjoint groups A and B , then $\mu_{A \cup B}(x) \geq \mu_{A' \cup B'}(x)$.

Under these conditions for both aggregation rules it make

A is said to be δ -decisive in competence over a group B ($A \cap B = \emptyset$) with competence β_B ($\beta_A \neq \beta_B$) if $\beta_{A \cup B} = \delta \cdot \beta_A + (1 - \delta) \cdot \beta_B$; a group A with opinion μ_A is said to be δ -decisive in opinion over a group B ($A \cap B = \emptyset$) with opinion μ_B and relative to a fixed alternative $x \in X$ ($\mu_A(x) \neq \mu_B(x)$) if

$$\mu_{A \cup B}(x) = \delta \cdot \mu_A(x) + (1 - \delta) \cdot \mu_B(x)$$

ETHICAL PROPERTIES OF THE WEIGHTED MEAN RULE

The following result has been proved in Montero (1988a), and it can be considered as an axiomatic justification for the weighted mean rules.

Theorem 1. Let us consider opinion aggregation rules verifying conditions (i'), (ii') and (iii'). Then, the minimum ratio opinion decisiveness $\delta/\beta(1)$ of individuals leads to the maximum for the weighted opinion aggregation rule such that

$$\mu_{A \cup B}(x) = (\text{card}(A) \cdot \beta_A \cdot \mu_A(x) + \text{card}(B) \cdot \beta_B \cdot \mu_B(x)) / (\text{card}(A \cup B) \cdot \beta_{A \cup B})$$

or any non-empty and disjoint groups $A, B \in G$, β being the aggregated weighted competence given by

$$\beta_{A \cup B} = (\text{card}(A) \cdot \beta_A + \text{card}(B) \cdot \beta_B) / \text{card}(A \cup B)$$

or any non-empty and disjoint groups $A, B \in G$. Moreover, this competence aggregation rule \otimes maximizes the minimum ratio competence decisiveness $\delta/\beta(1)$ of individuals, and it verifies conditions (i), (ii) and (iii).

It must be pointed out that the concept of an aggregation rule considered here generalizes an analogous aggregation operation of Pung and Fu (1975), avoiding a restrictive result obtained by them (and thus we can see that the main cause of such restrictive result is just that their aggregation rule does not depend on the size of the aggregated groups). Other axioms will lead to different aggregation rules, including those "mixed" rules of Pung and Fu (see Montero, 1988b, for an analysis in the context of multicriteria aggregation, and Dubois and Prade, 1985, for a relation with various fuzzy set aggregation connectives).

In any case, the weighted aggregation rule of Theorem 1 verifies ethical conditions translated into this context from classical impossibility theorems: the universal domain has been assured by definition, the anonymity will be assured due to condition (i'), the unanimity will be assured by condition ii'), the non-negative response and independence of irrelevant alternatives will hold due to condition (iii') and even the neutrality holds. Some formal definitions of all these ethical conditions are written below. Alternative definitions can be found in the literature (see, e.g., Dubois and Koning, 1989).

groups such that $A \cap B = A' \cap B' = \emptyset$, with $\beta(A) = \beta(A')$, $\beta(B) = \beta(B')$ and $\text{card}(A) = \text{card}(A')$, $\text{card}(B) = \text{card}(B')$. If $\mu_A \neq \mu_{A'}$ and $\mu_B \neq \mu_{B'}$ hold, then

$$(\mu_A \cdot A) * (\mu_B \cdot B) = (\mu_{A \cup B} \cdot A \cup B)$$

$$(\mu_{A'} \cdot A') * (\mu_{B'} \cdot B') = (\mu_{A' \cup B'} \cdot A' \cup B')$$

must verify that $\mu_{A \cup B} = \mu_{A' \cup B'}$.

Independence of Irrelevant Alternatives: Let $A, B \in \mathcal{P}(G)$ be disjoint and non-empty groups and let us suppose that

$$\mu_A(x) = \mu_{A'}(x) \quad \forall x \in Y \subset X, \quad \mu_B(x) = \mu_{B'}(x) \quad \forall x \in Y \subset X$$

Then

$$(\mu_A \cdot A) * (\mu_B \cdot B) = (\mu_{A \cup B} \cdot A \cup B)$$

$$(\mu_{A'} \cdot A') * (\mu_{B'} \cdot B') = (\mu_{A' \cup B'} \cdot A' \cup B')$$

must verify that

$$\mu_{A \cup B}(x) = \mu_{A' \cup B'}(x) \quad \forall x \in Y \subset X$$

Neutrality: Let $A, B \in \mathcal{P}(G)$ be disjoint and non-empty groups, and $P: X \rightarrow X$ be an arbitrary one-to-one mapping. If for each opinion $\mu \in \mathcal{F}(X)$ we define μ^P such that

$$\mu^P(x) = \mu(P(x)) \quad \forall x \in X$$

then

$$(\mu_A^P \cdot A) * (\mu_B^P \cdot B) = (\mu_{A \cup B}^P \cdot A \cup B)$$

Non-Negative Response: If $A, B \in \mathcal{P}(G)$ are disjoint and non-empty groups and

$$\mu_A(x) \geq \mu_{A'}(x) \quad \forall x \in X, \quad \mu_B(x) \geq \mu_{B'}(x) \quad \forall x \in X$$

with at least one strict inequality, then

$$(\mu_A \cdot A) * (\mu_B \cdot B) = (\mu_{A \cup B} \cdot A \cup B), \quad (\mu_{A'} \cdot A') * (\mu_{B'} \cdot B') = (\mu_{A' \cup B'} \cdot A' \cup B')$$

ought to verify

$$\mu_{A \cup B}(x) \geq \mu_{A' \cup B'}(x) \quad \forall x \in X$$

Unanimity: Let $A, B \in \mathcal{P}(G)$ be disjoint and non-empty groups with identical fuzzy preferences $\mu_A = \mu_B = \mu \in \mathcal{F}(X)$. Then

$$(\mu_A \cdot A) * (\mu_B \cdot B) = (\mu_{A \cup B} \cdot A \cup B)$$

Analogous ethical conditions (depending only on the size of the aggregated groups, as shown in Montero, 1985) could be imposed to competence aggregation operations \otimes , and it is easy to see that they are satisfied by weighted competence aggregation of Theorem 1.

Finally, we must point out a clear criticism to this approach, since it is assumed that the assignments are commensurable, and furthermore, if $\mu_i(x) > \mu_j(x)$ it is understood that individual i prefers alternative x more than individual j does. The problem of interpersonally comparing preference intensities is, as pointed out by Pattanaik (1971), part of the broader and controversial problem of the possibility of knowing other minds: but though in the present

sentation on the real hyper-space of the set of alternatives is obviously not necessarily unique. Hence, if real representations in a convex subset X of \mathbb{R}^n exist, first look for one of them where all the individual ones are concave. And it has been pointed out that the existence of such concave-compatible representation is not needed for arbitrary profiles of individual fuzzy preferences (trivial examples can be given by considering λ -peaked intensity preferences with constant intensities λ extreme alternatives).

Moreover, in practice only a finite number of alternatives can be evaluated at the first step. Some reasonable real representations could then be considered in order to initially search for better alternatives: for example, if alternatives are evaluated according to two characteristics, one represented by a real number between 0 and 10, the individual intensities could be initially estimated only for alternatives $\{(2,j,2,k)/j,k=0,1,\dots,5\}$, that is, for 36 alternatives covering as a net the whole space of alternatives $[0,10] \times [0,10]$; if concavity holds for this real representation of individual preferences, then a best alternative must be looked for around the peaks obtained by weighted mean aggregation; for example, if only one λ is reached in alternative (4,6), it will be enough to iter in the next step only the subspace of alternatives $\leq [5,7]$, where another net of points could be established in order to repeat the procedure. If concavity does not hold, as a concave representation could be tried by considering appropriate order preserving one-to-one mappings of the initial evaluation space $[0,10]$ of each characteristic into different real intervals, or perhaps by considering a deeper transformation (alternatives could be indexed in terms of different but more appropriate characteristics).

REFERENCES

- Arrow (1951, 1964), *Social Choice and Individual Values*, Wiley, New York.
- Black (1958), *The Theory of Committees and Elections*, Cambridge University Press, Cambridge.
- Cholewa (1985), *Aggregation of Fuzzy Opinions: An Axiomatic Approach*, *Fuzzy Sets Syst.* 17, 249-258.
- Cholewa and J.L. Koning (1989), *Social Choice Axioms for Fuzzy Sets Aggregation*, *Fuzzy Sets Syst.* (to appear).
- Cholewa and H. Prade (1985), *A review of Fuzzy sets aggregation Connectives*, *Inf. Sci.* 36, 85-121.
- Fung and K.S. Fu (1975), *An Axiomatic Approach to Rational Decision Making in a Fuzzy Environment*; in: L. Alchourrón, K.S. Fu, K. Tanaka and M. Shimura (Eds.), *Fuzzy Sets and their Applications to Cognitive and Decision Processes*, Academic Press, New York, pp. 227-256.

32, 525-531.

- J. Montero (1985), *A note on Fung-Fu's Theorem*, *Fuzzy Sets Syst.* 17, 259-269.
- J. Montero (1988a), *Aggregation of Fuzzy Opinions in a Non-Homogeneous Group*, *Fuzzy Sets Syst.* 25, 15-20.
- J. Montero (1988b), *An Axiomatic Approach to Fuzzy Multicriteria Analysis*; in: M.M. Gupta and T. Yamakawa (Eds.), *Fuzzy Logic in Knowledge-Based Systems, Decision and Control*, North-Holland, Amsterdam, pp. 259-269.
- J. Montero (1989), *Weighted Aggregation and Single Peaked Intensities*, *Workshop on Aggregation and Best Choices on Imprecise Opinions*, Brussels.
- P.K. Pattanaik (1971), *Voting and Collective Choice*, Cambridge University Press, Cambridge.
- A.K. Sen (1970), *Collective Choice and Social Welfare*, Holden-Day, San Francisco.
- J.C. Vansnick (1987), *Intensity of Preference*; in: Y. Sawaragi, K. Inoue and H. Nakayama (Eds.), *Toward Interactive and Intelligent Decision Support Systems* (volume 2), Springer-Verlag, Berlin, pp. 220-229.

method of measuring and interpersonally comparing the intensities of individual preferences, it can hardly be denied that ethically it is desirable to take them into account. An interesting attempt in this sense is that of Ansnick (1986).

- CONCAVITY OF INDIVIDUAL PROFILES

In our context, single-peakedness means that alternatives are ordered in the real hyper-space R^k of k characteristics such a way that, as we go from left to the right in each characteristic, every individual preferences increases up to a peak, and then decreases after we pass it. As shown in Montero (1989), such a property will hold when the considered intensity is concave from an analytic point of view:

Definition 1. Let $\mu: X \rightarrow [0,1]$ be a fuzzy set of feasible alternatives defined over a convex subset of the real per-space $X \subset R^k$. Then μ is said to be "concave" if

$$\mu(\lambda \cdot x + (1-\lambda) \cdot y) \geq \lambda \cdot \mu(x) + (1-\lambda) \cdot \mu(y) \quad \forall x, y \in X, \forall \lambda \in [0,1]$$

It is clear that a single-peaked representation on the real line is in its spirit close to Black's single-peakedness and even to Inada's single-peakedness (Inada, 1964), a condition less restrictive than single-peakedness which allows for indifference plateaus, and also assuring a consensus transitive solution under the crisp majority rule. Though it is clear that single-peaked intensity and single-caved intensity are not always concave, the introduction of concavity is justified due to some interesting properties which will appear when individual intensities are concave compatible in our context.

Definition 2. A profile of individual intensities $\mu: X \rightarrow [0,1]$, $i \in G$ defined over a convex set $X \subset R^k$ of alternatives verifies the property of "concavity" if they are all concave.

In this way, if concavity is assumed, an important but desirable restriction on the family of admissible preference patterns has been introduced, since each individual preference must be inside the set $\mathcal{C}(X) \subset \mathcal{F}(X)$ of concave functions. Therefore, if it is considered that the restrictions on individual preferences must be just the restrictions on social preferences, our aggregation rule \star must be defined as associative and commutative correspondence

$$\star: (\mathcal{C}(X) \times \mathcal{P}(G)) \times (\mathcal{C}(X) \times \mathcal{P}(G)) \rightarrow (\mathcal{C}(X) \times \mathcal{P}(G))$$

verifying the above ethical conditions.

It must be pointed out that single-peakedness of individual intensities does not assure a social single-peaked union under the weighted mean rule. Such a closure theorem can be assured under concavity (its mathematical proof is trivial).

Theorem 2. Let us assume that the concavity property holds, and let us consider the weighted aggregation rule as given in

disjoint and non-empty groups $A, B \in \mathcal{P}(G)$.

Concavity of a social preference pattern can be understood as a condition which assures an equilibrium and stability in group decision making: a best alternative is around one point, and a manipulation or measurement errors (if they are not too big) can modify in fact the solution, but new solution will not be in any case too far from the initial solution.

The concave individual intensities mean in fact that people are inclined to reach consensus. The single-peakedness of individual intensities seems to be a natural assumption when dealing with single characteristic problems, but if they are non-concave, they represent very clear individual preferences, with people rejecting other alternatives different than their own best alternatives: binary intensities (those verifying $\mu(x) \in \{0,1\} \quad \forall x \in X$) will never be concave unless they are constant (that is, $\mu(x) = \mu(y) \quad \forall x, y \in X$). For example, if there is an individual $i \in G$ such that $\mu(x_i) = 1$ but $\mu(y) = 0, \quad \forall y \neq x_i$, for a fixed alternative $x_i \in X$, our common sense

tells us that a satisfactory consensory solution will be very difficult due to such a crisp (absolutely clear) opinion, and in fact there is no representation making such a preference concave. In some way we can conclude that Theorem 2 reveals how "spreaded" intensities (in the sense of concavity) can easily be aggregated according to the weighted mean rule, and points out the expected difficulties in aggregating crisp (or too clear) opinions.

In any case, it also must be pointed out that concavity requires a sensitivity of the decision maker, since around best alternatives the intensity must be strictly decreasing (there is no identical intensities at each side of the peak).

A dual result can be obtained when the values of our intensity preferences μ are understood as degrees of rejecting each alternative by the individuals (like a fuzzy veto instead of a fuzzy preference). It shows the difficulties in aggregating vetoes of an arbitrary shape, and its mathematical proof is also trivial.

Theorem 3. Let us consider the weighted aggregation rule \star as given in Theorem 1, and let us suppose that each individual opinion verifies convexity, that is,

$$\mu(\lambda \cdot x + (1-\lambda) \cdot y) \leq \lambda \cdot \mu(x) + (1-\lambda) \cdot \mu(y) \quad \forall x, y \in X, \forall \lambda \in [0,1]$$

Then, an aggregated opinion for any pair of disjoint and non-empty groups in G also verifies convexity.

Under the concavity condition, the weighted aggregation will be between moderate alternatives (that is, non-extreme alternatives). Such a property seems in principle desirable for the stability of any social system.

S.- FINAL COMMENTS

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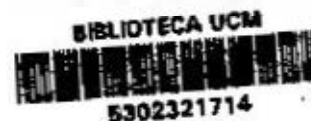
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